EIGHTH EDITION

Algebra & Trigonometry

RICHARD N. AUFMANN | RICHARD D. NATION

Matej Pribelsky/Getty Images

EIGHTH EDITION

ALGEBRA AND TRIGONOMETRY

Richard N. Aufmann
Richard D. Nation

CENGAGE Learning®

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

***Algebra and Trigonometry*, Eighth Edition**
Richard N. Aufmann, Richard D. Nation

Product Director: Liz Covello

Product Manager: Gary Whalen

Content Developer: Stacy Green

Content Coordinator: Cynthia Ashton

Product Assistant: Samantha Lugtu

Media Developer: Lynh Pham

Brand Manager: Gordon Lee

Market Development Manager: Mark Linton

Content Project Manager: Jennifer Risden

Art Director: Vernon Boes

Manufacturing Planner: Becky Cross

Rights Acquisitions Specialist: Tom McDonough

Production Service: MPS Limited

Photo Researcher: Bill Smith Group

Text Researcher: Bill Smith Group

Copy Editor: Susan Reiland

Illustrator: Network Graphics; MPS Limited

Text Designer: Diane Beasley

Cover Designer: Lisa Henry

Cover Images: Matej Pribelsky/Getty Images; Vibrant Image Studio/Shutterstock.com

Compositor: MPS Limited

WCN: 02-200-203

Library of Congress Control Number: 2013940727

ISBN-13: 978-1-285-44942-5

ISBN-10: 1-285-44942-8

Cengage Learning
200 First Stamford Place, 4th Floor
Stamford, CT 06902
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at **www.cengage.com/global**.

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Cengage Learning Solutions, visit **www.cengage.com**.

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com**.

Printed in the United States of America
1 2 3 4 5 6 7 17 16 15 14 13

Contents

CHAPTER 3 Polynomial and Rational Functions 259

CHAPTER 4 Exponential and Logarithmic Functions 335

CHAPTER 5 Trigonometric Functions 429

CHAPTER 6 Trigonometric Identities, Inverse Functions, and Equations 519

CHAPTER 7 Applications of Trigonometry 587

CHAPTER 8 Topics in Analytic Geometry 643

CHAPTER 9 Systems of Equations and Inequalities 731

CHAPTER 10 Matrices 795

CHAPTER 11 Sequences, Series, and Probability 861

Preface

We are proud to offer the eighth edition of *Algebra and Trigonometry*. Your success in algebra and trigonometry is important to us. To guide you to that success, we have created a textbook with features that promote learning and support various learning styles. These features are highlighted below. We encourage you to examine the features and use them to help you successfully complete this course.

Features

Chapter Openers

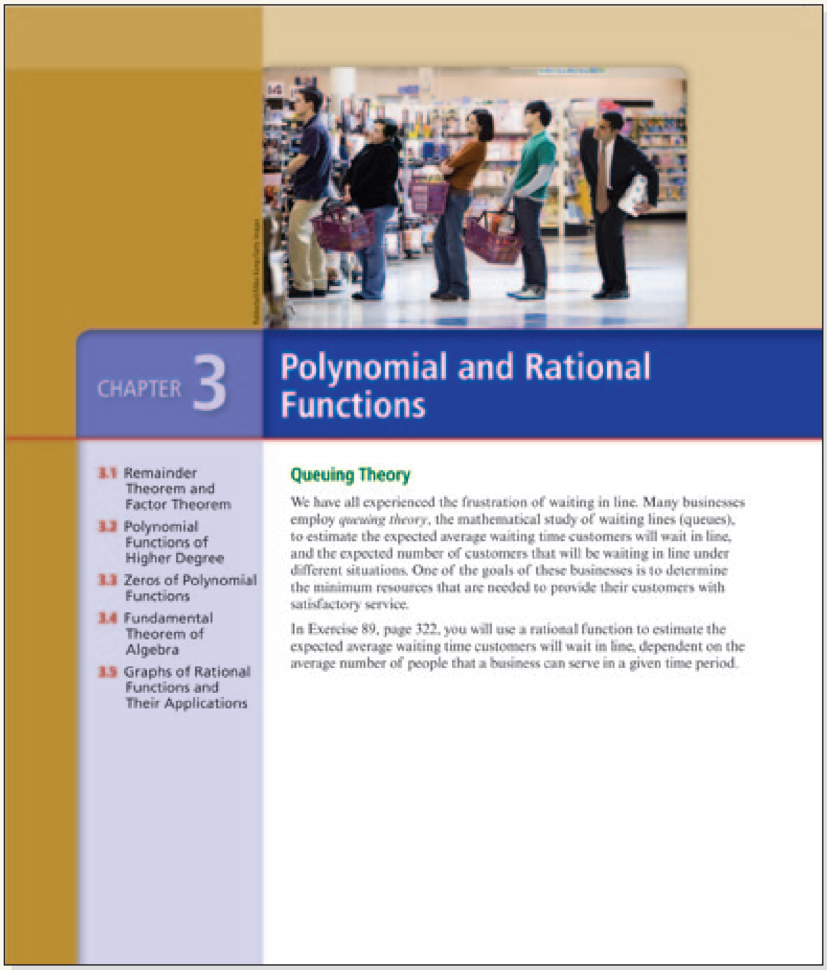
Each Chapter Opener demonstrates a contemporary application of a mathematical concept developed in that chapter.

Related Exercise References

Each Chapter Opener cites a particular exercise within the chapter that is related to the chapter opener topic.

Listing of Major Concepts

A list of major concepts in each section is provided in the margin of the first page of each section.

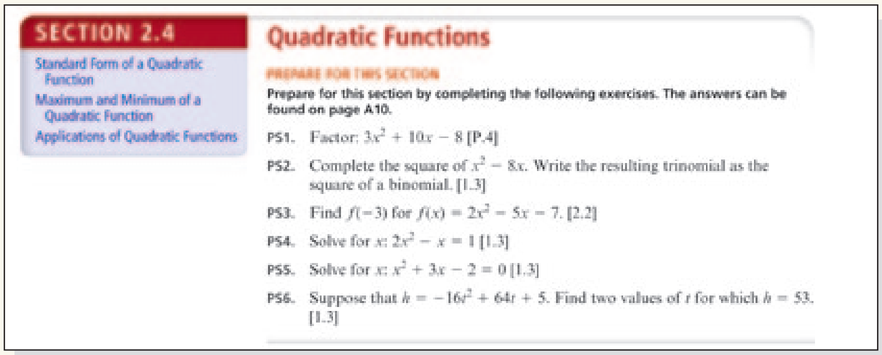


CHAPTER 3 Polynomial and Rational Functions

Queuing Theory

We have all experienced the frustration of waiting in line. Many businesses employ *queuing theory*, the mathematical study of waiting lines (queues), to estimate the expected average waiting time customers will wait in line, and the expected number of customers that will be waiting in line under different situations. One of the goals of these businesses is to determine the minimum resources that are needed to provide their customers with satisfactory service.

In Exercise 89, page 322, you will use a rational function to estimate the expected average waiting time customers will wait in line, dependent on the average number of people that a business can serve in a given time period.



SECTION 2.4

Standard Form of a Quadratic Function
Maximum and Minimum of a Quadratic Function
Applications of Quadratic Functions

Quadratic Functions

PREPARE FOR THIS SECTION

Prepare for this section by completing the following exercises. The answers can be found on page A10.

PS1. Factor: $3x^2 + 10x - 8$ [P.4]

PS2. Complete the square of $x^2 - 8x$. Write the resulting trinomial as the square of a binomial. [1.3]

PS3. Find $f(-3)$ for $f(x) = 2x^2 - 5x - 7$. [2.2]

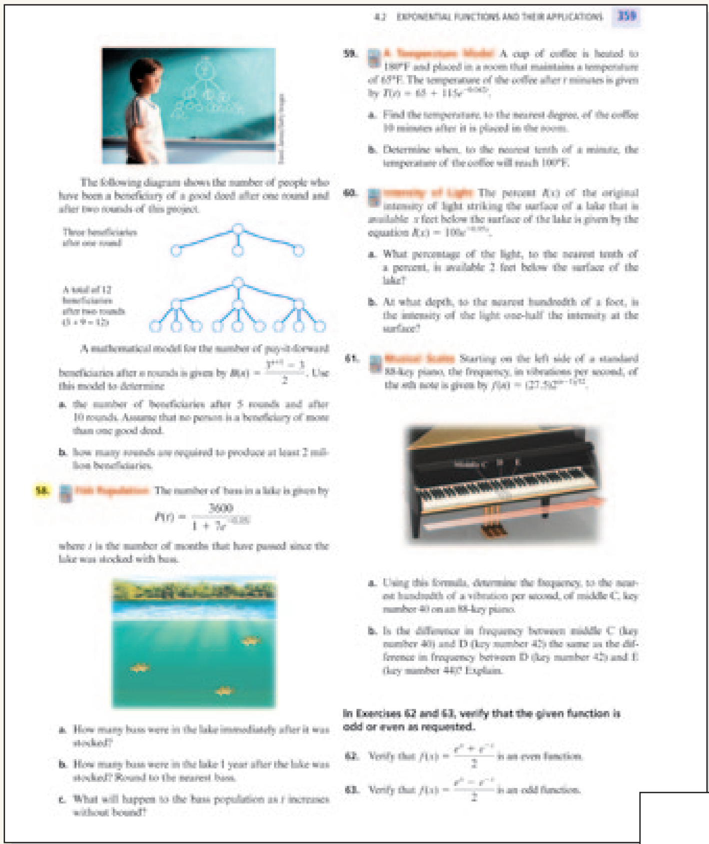
PS4. Solve for x: $2x^2 - x = 1$ [1.3]

PS5. Solve for x: $x^2 + 3x - 2 = 0$ [1.3]

PS6. Suppose that $h = -16t^2 + 64t + 5$. Find two values of t for which $h = 53$. [1.3]

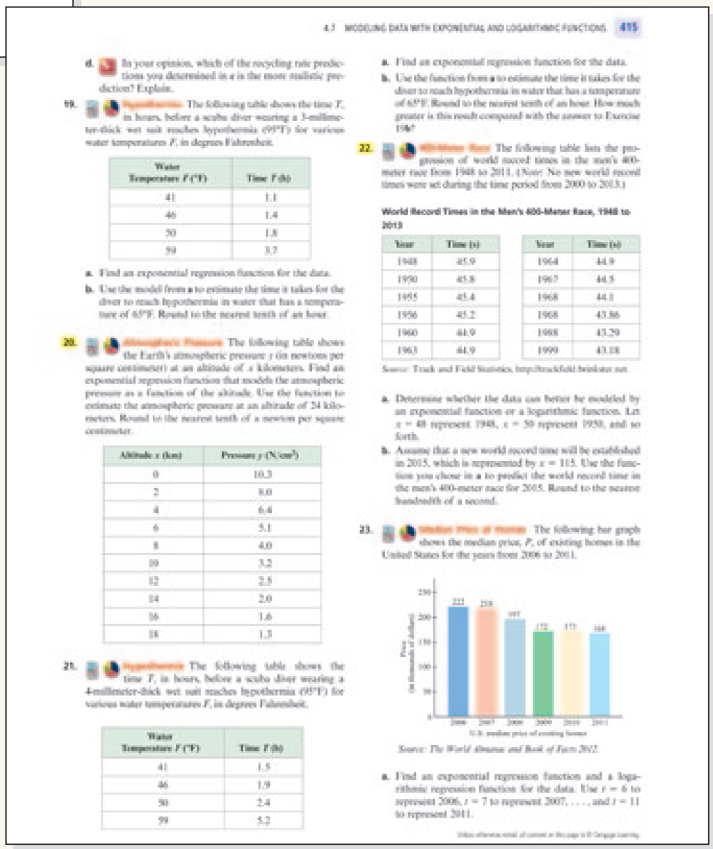
Prepare for This Section

Each section (after the first section) of a chapter opens with review exercises titled Prepare for This Section. These exercises give you a chance to test your understanding of prerequisite skills and concepts before proceeding to the new topics presented in the section.



Thoughtfully Designed Exercise Sets

We have thoroughly reviewed each exercise set to ensure a smooth progression from routine exercises to exercises that are more challenging. The exercises illustrate the many facets of topics discussed in the text. The exercise sets emphasize skill building, skill maintenance, conceptual understanding, and, as appropriate, applications. Each chapter includes a Chapter Review Exercise set and each chapter, except Chapter P, includes a Cumulative Review Exercise set.



Contemporary Applications

Carefully developed mathematics is complemented by abundant, relevant, and contemporary applications. Many of these feature real data, tables, graphs, and charts. Applications demonstrate the value of algebra and cover topics from a wide variety of disciplines. Besides providing motivation to study mathematics, the applications will help you develop good problem-solving skills.

By incorporating many interactive learning techniques, including the key features outlined below, *Algebra and Trigonometry* uses the proven Aufmann Interactive Methods (AIM) to help you understand concepts and obtain greater mathematical fluency. The AIM consists of **Annotated Examples** followed by **Try Exercises** (and solutions) and a conceptual **Question/Answer** follow-up. See the samples below:

EXAMPLE 5 Calculate the Airtime for a Snowboarder's Jump

The height $h(t)$, in feet, of a snowboarder t seconds after beginning a certain jump can be approximated by $h(t) = -16t^2 + 22.9t + 9$. If the snowboarder lands at a point that is 3 feet below the base of the jump, determine the *airtime* (the time the snowboarder is in the air) for this jump. Round to the nearest tenth of a second.

Solution

Because $h(t)$ represents the height of the snowboarder t seconds after the beginning of the jump, the snowboarder lands when $h(t) = -3$, 3 feet *below* the base of the jump.

$$h(t) = -16t^2 + 22.9t + 9$$
$$-3 = -16t^2 + 22.9t + 9 \qquad \text{• Replace } h(t) \text{ with } -3.$$
$$0 = -16t^2 + 22.9t + 12$$
$$t = \frac{-22.9 \pm \sqrt{22.9^2 - 4(-16)(12)}}{2(-16)} \qquad \text{• Use the quadratic formula.}$$
$$= \frac{-22.9 \pm \sqrt{1292.41}}{-32}$$
$$\approx -0.4 \quad \text{or} \quad 1.8 \qquad \text{• Use a calculator.}$$

Because a negative time is not possible, the airtime for this jump is approximately 1.8 seconds.

▶ Try Exercise 48, page 207

◀ Engaging Examples

Examples are designed to capture your attention and help you master important concepts.

◀ Annotated Examples

Step-by-step solutions are provided for most numbered examples.

▲ Try Exercises

A reference to an exercise follows all worked examples. This exercise provides you with an opportunity to test your understanding of concepts by working an exercise related to the worked example.

▶ Solutions to Try Exercises

Complete solutions to the Try Exercises can be found in the Solutions to the Try Exercises appendix.

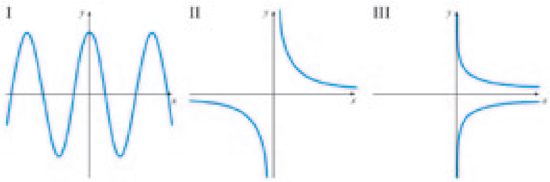
Exercise Set 2.4, page 206

48. The soccer ball hits the ground when $h(t) = 0$.

$$h(t) = -4.9t^2 + 12.8t$$
$$0 = -4.9t^2 + 12.8t \qquad \text{• Replace } h(t) \text{ with } 0.$$
$$0 = t(-4.9t + 12.8) \qquad \text{• Solve for } t.$$
$$t = 0 \quad \text{or} \quad t = \frac{-12.8}{-4.9} \approx 2.6$$

The soccer ball hits the ground in approximately 2.6 seconds.

Question • Which of the graphs below, I, II, or III, is **a.** symmetric with respect to the x-axis? **b.** symmetric with respect to the y-axis?



Answer • **a.** III is symmetric with respect to the x-axis. **b.** I is symmetric with respect to the y-axis.

◀ Question/Answer

In each section, we have posed at least one question that encourages you to pause and think about the concepts presented in the current discussion. To ensure that you do not miss this important information, the answer is provided as a footnote on the same page.

Zero of a Function

A value a in the domain of a function f for which $f(a) = 0$ is called a **zero** of f.

EXAMPLES

- Let $f(x) = 2x - 4$. When $x = 2$, we have

$$f(x) = 2x - 4$$
$$f(2) = 2(2) - 4$$
$$= 0$$

Because $f(2) = 0$, 2 is a zero of f.

Immediate Examples of Definitions and Concepts

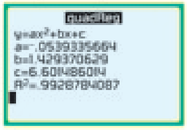
Immediate examples of many definitions and concepts are provided to enhance your understanding of new topics.

Margin Notes alert you to a point requiring special attention or are used to provide study tips.

Note

For nonlinear regression calculations, the value of r is not shown on a TI-83/TI-83 Plus/TI-84 Plus graphing calculator. In such cases, the coefficient of determination is used to determine how well the data fit the model.

2. Find the regression equation using QuadReg in the STAT CALC menu. For a TI-83/TI-83 Plus/TI-84 Plus calculator, press STAT ▶ 5 ENTER. Scroll to Calculate and press ENTER.



To Review Notes in the margin will help you recognize the prerequisite skills needed to understand new concepts. These notes direct you to the appropriate page or section for review.

TO REVIEW
Pythagorean Theorem
See pages 102–103.

The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in Figure 2.5 is the length of the hypotenuse of a right triangle whose sides are horizontal and vertical line segments that measure $|x_2 - x_1|$ and $|y_2 - y_1|$, respectively. Applying the Pythagorean Theorem to this triangle produces

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Calculus Connection Icons identify topics that will be revisited in a subsequent calculus course.

CALCULUS CONNECTION

Difference Quotient

The expression

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

is called the **difference quotient** of f. It enables us to study the manner in which a function changes in value as the independent variable changes.

EXAMPLE 7 Solve a Logarithmic Equation

Solve: $\ln(3x + 8) = \ln(2x + 2) + \ln(x - 2)$

Algebraic Solution

$\ln(3x + 8) = \ln(2x + 2) + \ln(x - 2)$

$\ln(3x + 8) = \ln[(2x + 2)(x - 2)]$ • Product property

$\ln(3x + 8) = \ln(2x^2 - 2x - 4)$

$3x + 8 = 2x^2 - 2x - 4$ • One-to-one property of logarithms

$0 = 2x^2 - 5x - 12$ • Subtract $3x + 8$ from each side.

$0 = (2x + 3)(x - 4)$ • Factor.

$x = -\frac{3}{2}$ or $x = 4$ • Solve for x.

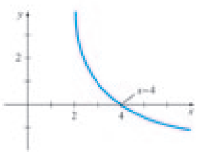
A check will show that 4 is a solution but that $-\frac{3}{2}$ is not a solution.

Visualize the Solution

The graph of

$y = \ln(3x + 8) - \ln(2x + 2) - \ln(x - 2)$

has only one x-intercept. Thus there is only one real solution.



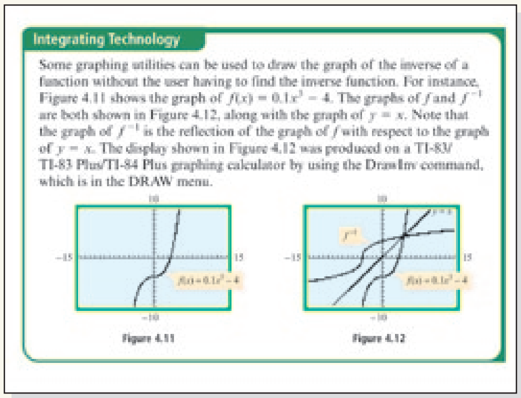
▶ Try Exercise 40, page 388

Visualize the Solution

When appropriate, both algebraic and graphical solutions are provided to help you visualize the mathematics of an example and to create a link between the algebraic and visual components of a solution.

Integrating Technology

Integrating Technology boxes show how technology can be used to illustrate concepts and solve many mathematical problems.

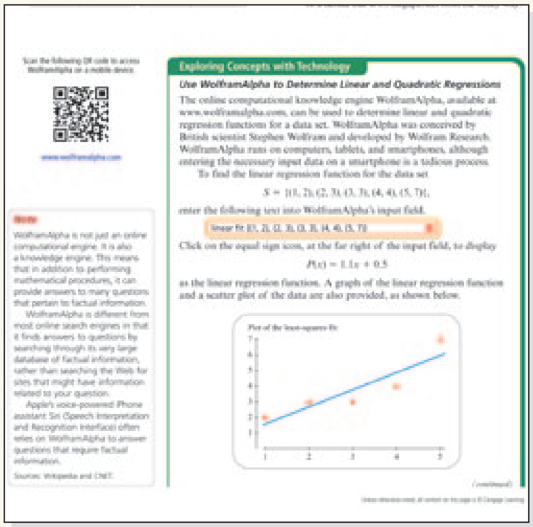

Integrating Technology

Some graphing utilities can be used to draw the graph of the inverse of a function without the user having to find the inverse function. For instance, Figure 4.11 shows the graph of $f(x) = 0.1x^3 - 4$. The graphs of f and f^{-1} are both shown in Figure 4.12, along with the graph of $y = x$. Note that the graph of f^{-1} is the reflection of the graph of f with respect to the graph of $y = x$. The display shown in Figure 4.12 was produced on a TI-83/TI-83 Plus/TI-84 Plus graphing calculator by using the DrawInv command, which is in the DRAW menu.

Figure 4.11 Figure 4.12

Exploring Concepts with Technology

The optional Exploring Concepts with Technology feature appears after the last section in each chapter and provides you the opportunity to use a calculator, a mobile device, or a computer to solve computationally difficult problems.


Scan the following QR code to access WolframAlpha on a mobile device.

www.wolframalpha.com

Exploring Concepts with Technology

Use WolframAlpha to Determine Linear and Quadratic Regressions

The online computational knowledge engine WolframAlpha, available at www.wolframalpha.com, can be used to determine linear and quadratic regression functions for a data set. WolframAlpha was conceived by British scientist Stephen Wolfram and developed by Wolfram Research. WolframAlpha runs on computers, tablets, and smartphones, although entering the necessary input data on a smartphone is a tedious process.

To find the linear regression function for the data set

$S = \{(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)\}$,

enter the following text into WolframAlpha's input field.

linear fit {1, 2}, {2, 3}, {3, 3}, {4, 4}, {5, 7}

Click on the equal sign icon, at the far right of the input field, to display

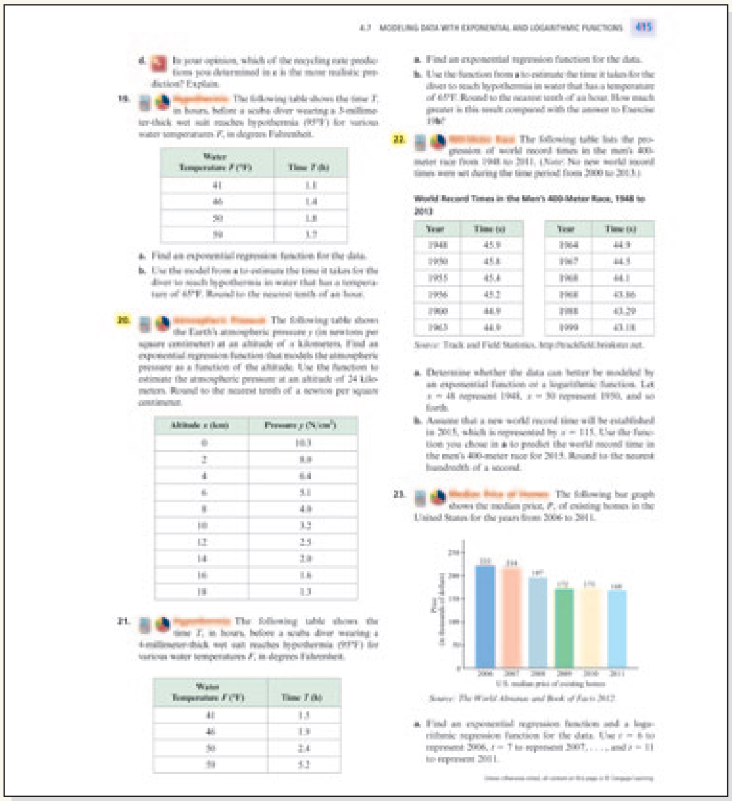
$P(x) = 1.1x + 0.5$

as the linear regression function. A graph of the linear regression function and a scatter plot of the data are also provided, as shown below.

(continued)

Modeling

Modeling sections and exercises rely on the use of a graphing calculator, a mobile device, or a computer. These optional sections and exercises introduce the idea of a mathematical model and help you see the relevance of mathematical concepts.


4.7 MODELING DATA WITH EXPONENTIAL AND LOGARITHMIC FUNCTIONS 415

Mid-Chapter Quizzes

The Mid-Chapter Quizzes help you assess your understanding of the concepts studied earlier in the chapter. The answers for all exercises in the Mid-Chapter Quizzes, along with a reference to the section in which a particular concept was presented, are provided in the Answers to Selected Exercises appendix.

MID-CHAPTER 4 QUIZ

1. Use composition of functions to verify that
$$f(x)=\frac{500+120x}{x} \quad \text{and} \quad g(x)=\frac{500}{x-120}$$
are inverses of each other.
2. Find the inverse of $f(x)=\frac{24x+5}{x-4}$, $x \neq 4$. State any restrictions on the domain of $f^{-1}(x)$.
3. Evaluate $f(x)=e^x$, for $x=-2.4$. Round to the nearest ten-thousandth.
4. Write $\ln x = 6$ in exponential form.
5. Graph $f(x)=\log_3(x+3)$.
6. Expand $\ln\left(\frac{xy^3}{e^2}\right)$. Assume x and y are positive real numbers.
7. Write $\log_3 x^4 - 2\log_3 z + \log_3(xy^2)$ as a single logarithm with a coefficient of 1. Assume all variables are positive real numbers.
8. Use the change-of-base formula to evaluate $\log_8 411$. Round to the nearest ten-thousandth.
9. What is the Richter scale magnitude of an earthquake with an intensity of $789{,}251I_0$? Round to the nearest tenth.
10. How many times as great is the intensity of an earthquake that measures 7.9 on the Richter scale than the intensity of an earthquake that measures 5.1 on the Richter scale?

CHAPTER 4 TEST PREP

The following test prep table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

4.1 Inverse Functions

Graph the Inverse of a Function A function f has an inverse function if and only if it is a one-to-one function. The graph of f and the graph of its inverse f^{-1} are symmetric with respect to the line given by $y = x$.	See Example 1, page 338, and then try Exercises 1 and 2, page 423.
Composition of Inverse Functions Property If f is a one-to-one function, then f^{-1} is the inverse function of f if and only if $(f \circ f^{-1})(x) = f[f^{-1}(x)] = x$ for all x in the domain of f^{-1} and $(f^{-1} \circ f)(x) = f^{-1}[f(x)] = x$ for all x in the domain of f.	See Example 2, page 339, and then try Exercises 3 and 6, page 423.

Chapter Test Preps

The Chapter Test Preps summarize the major concepts discussed in each chapter. These Test Preps help you prepare for a chapter test. For each concept, there is a reference to a worked example illustrating the concept and at least one exercise in the Chapter Review Exercise set relating to that concept.

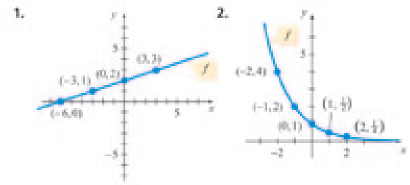
Chapter Review Exercise Sets and Chapter Tests

The Chapter Review Exercise sets and the Chapter Tests at the end of each chapter are designed to provide you with another opportunity to assess your understanding of the concepts presented in a chapter. The answers for all exercises in the Chapter Review Exercise sets and the Chapter Tests, along with a reference to the section in which the concept was presented, are provided in the Answers to Selected Exercises appendix.

CHAPTER 4 REVIEW EXERCISES

In Exercises 1 and 2, draw the graph of the inverse of the given function.

1. 2.



In Exercises 3 to 6, use composition of functions to determine whether the given functions are inverse functions.

In Exercises 25 to 36, sketch the graph of each function.

25. $f(x)=(2.5)^x$
26. $f(x)=\left(\frac{1}{4}\right)^x$
27. $f(x)=3^{|x|}$
28. $f(x)=4^{-|x|}$
29. $f(x)=2^x-3$
30. $f(x)=2^{(x-3)}$
31. $f(x)=\log_5 x$
32. $f(x)=\log_{1/3} x$
33. $f(x)=\frac{1}{3}\log x$
34. $f(x)=3\log x^{1/3}$
35. $f(x)=-\frac{1}{2}\ln x$
36. $f(x)=-\ln|x|$

CHAPTER 4 TEST

1. Find the inverse of $f(x)=2x-3$. Graph f and f^{-1} on the same coordinate axes.
2. Find the inverse of $f(x)=\frac{x}{4x-8}$, where the domain of f is $\{x \mid x > 2\}$. State the domain and the range of f^{-1}.
3. a. Write $\log_b(5x-3)=c$ in exponential form.
   b. Write $3^{x/2}=y$ in logarithmic form.
4. Expand $\log_b \frac{z^2}{y^3\sqrt{x}}$.
5. Write $\log(2x+3)-3\log(x-2)$ as a single logarithm with a coefficient of 1.
6. Use the change-of-base formula and a calculator to approximate $\log_4 12$. Round your result to the nearest ten-thousandth.

New Features in This Eighth Edition!

NEW Concept Check Exercises

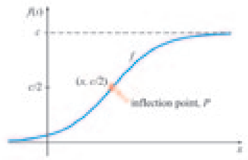
Each exercise set starts with exercises that are designed to test your understanding of new concepts.

> **EXERCISE SET 4.5**
>
> **Concept Check**
>
> **1.** Some exponential equations can be solved by using the Equality of Exponents Theorem. What is the Equality of Exponents Theorem?
>
> **2.** Name two methods that can be used to estimate the solutions of an equation of the form $f(x) = g(x)$, with the aid of a graphing utility.

NEW Enrichment Exercises

Each exercise set concludes with an exercise or exercises designed to extend the concepts presented in the section or to provide exercises that challenge your problem-solving abilities.

> **Enrichment Exercises**
>
> **35.** **Power Functions** A function that can be written in the form $y = ax^b$ is said to be a **power function.** Some data sets can best be modeled by a power function. On a TI-83/84 calculator, the PwrReg instruction is used to produce a power regression function for a set of data. Find the power regression function for the following data.
>
> | x | 1 | 2 | 3 | 4 | 5 | 6 |
> |---|---|---|---|---|---|---|
> | y | 2.1 | 5.5 | 9.8 | 14.6 | 20.1 | 25.8 |
>
> **36.** **Period of a Pendulum** The following table shows the time t (in seconds) of the period of a pendulum of length l (in feet). (*Note:* The period of a pendulum is the time it takes the pendulum to complete a swing from the right to the left and back.)
>
> | Length l (ft) | 1 | 2 | 3 | 4 | 6 | 8 |
> |---|---|---|---|---|---|---|
> | Time t (s) | 1.11 | 1.57 | 1.92 | 2.25 | 2.72 | 3.14 |
>
> Use the power regression function for the data to estimate the
>
> with domain the set of real numbers, is its shape. The graph of every logistic function has an S-shape and a single inflection point, which separates the graph into two equal regions of opposite concavity. For instance, in the following graph f is concave upward to the left of its inflection point P and it is concave downward to the right of P.

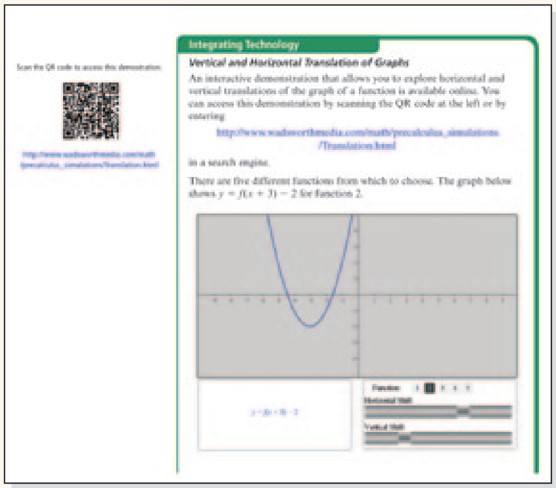


> It is easy to identify the y-coordinate of the inflection point P, because the graph of f is symmetrical about its inflection point. Thus the inflection point must occur halfway up the graph at a height of $y = c/2$.
>
> Determine the x-coordinate of the inflection point of f.

NEW Interactive Demonstrations

Our new Interactive Demonstrations allow you to adjust parameters and immediately see the change produced by your adjustment. These Interactive Demonstrations run on computers and on mobile devices. Instructions for using each of the demonstrations are provided.

You can access these Interactive Demonstrations by scanning a QR code with a QR reader app or by using the Web address listed below the QR code.



> Scan the following QR code to access WolframAlpha on a mobile device.
>
> www.wolframalpha.com
>
> **Note**
>
> WolframAlpha is not just an online computational engine. It is also a knowledge engine. This means that in addition to performing mathematical procedures, it can provide answers to many questions that pertain to factual information.
>
> **Exploring Concepts with Technology**
>
> ***Use WolframAlpha to Determine Linear and Quadratic Regressions***
>
> The online computational knowledge engine WolframAlpha, available at www.wolframalpha.com, can be used to determine linear and quadratic regression functions for a data set. WolframAlpha was conceived by British scientist Stephen Wolfram and developed by Wolfram Research. WolframAlpha runs on computers, tablets, and smartphones, although entering the necessary input data on a smartphone is a tedious process.
>
> To find the linear regression function for the data set
>
> $$S = \{(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)\},$$
>
> enter the following text into WolframAlpha's input field.
>
> linear fit {(1, 2), (2, 3), (3, 3), (4, 4), (5, 7)}
>
> Click on the equal sign icon, at the far right of the input field, to display
>
> $$P(x) = 1.1x + 0.5$$
>
> as the linear regression function. A graph of the linear regression function and a scatter plot of the data are also provided, as shown below.

NEW An Alternative Technology Approach via WolframAlpha

Several Integrating Technology features and some Exploring Concepts with Technology features show how WolframAlpha can be used to perform computations, solve equations, graph functions, and find regression functions. WolframAlpha, which runs on computers and mobile devices, often provides an alternative to a graphing calculator. WolframAlpha can be accessed by scanning a QR code or by using the address www.wolframalpha.com.

In addition to the new features, the following changes appear in this eighth edition of *Algebra and Trigonometry*.

Chapter P **Preliminary Concepts**

- A new chapter opener introduces some of the concepts in this chapter.
- **P.1** New application exercises and Enrichment Exercises have been added.
- **P.2** One example has been revised and Enrichment Exercises have been added.
- **P.3** One example has been revised.
- **P.4** Enrichment Exercises have been added.
- **P.5** A continued fraction exercise has been added to the Enrichment Exercises.
- **P.6** Enrichment Exercises have been added.

Chapter 1 **Equations and Inequalities**

- A new chapter opener introduces some of the concepts in this chapter.
- **1.1** New application exercises and Enrichment Exercises have been added.
- **1.2** Three examples have been updated and a new example has been added. Several exercises have been revised and Enrichment Exercises have been added.
- **1.3** New application exercises and Enrichment Exercises have been added.
- **1.4 and 1.5** An Enrichment Exercise has been added.

Chapter 2 **Functions and Graphs**

- **2.2** The introduction to piecewise-defined functions has been expanded and the example has been revised. New exercises involving piecewise-defined functions have been added.
- **2.3** A new example has been added and several exercises have been updated.
- **2.4** The business application example has been revised and application exercises have been added.
- **2.5** Three new Interactive Demonstrations have been added. They illustrate translations, reflections, and compressing and stretching of graphs. Enrichment Exercises have been added.
- **2.7** Two exercises have been updated and a new application exercise has been added.
- The Exploring Concepts with Technology feature has been revised to illustrate how WolframAlpha can be used to find linear and quadratic regression functions.

Chapter 3 **Polynomial and Rational Functions**

- A new chapter opener introduces some of the concepts in this chapter.
- **3.2** The Integrating Technology feature has been expanded to include instructions on how to use WolframAlpha to graph, find extrema, and evaluate polynomial functions. The cubic regression example has been updated and new application exercises have been added.
- **3.3 and 3.4** One example has been revised.
- **3.5** An application exercise and an exercise that involves the parabolic asymptote of a rational function have been added.
- The Exploring Concepts with Technology feature has been revised to illustrate how WolframAlpha can be used to find zeros of a polynomial function and to find cubic and quartic regression functions.
- Several exercises in the Review Exercises and the Chapter Test have been updated.

Chapter 4 **Exponential and Logarithmic Functions**

- A new chapter opener introduces some of the concepts in this chapter.
- **4.2** A new Integrating Technology feature illustrates how to use WolframAlpha to solve exponential equations. A new exercise that demonstrates the rapid growth of an exponential function has been included in the Enrichment Exercises.
- **4.3** Several exercises have been revised and two application exercises have been added.
- **4.4** A new Integrating Technology feature illustrates how to use WolframAlpha to evaluate logarithms with various bases. Several examples and exercises have been updated.
- **4.6** Several population growth and compound interest exercises have been updated.

- **4.7** Two examples and several exercises have been updated.
- The Exploring Concepts with Technology feature has been revised to illustrate how WolframAlpha can be used to find exponential and logarithmic regression functions.
- Several exercises in the Review Exercises and the Chapter Test have been updated.

Chapter 5 **Trigonometric Functions**

- A new chapter opener introduces some of the concepts in this chapter.
- **5.1** Two examples have been revised and a new application example has been added. Application exercises involving arc length and angular speed have been added.
- **5.2** A new example and several application exercises have been added.
- **5.3** A new example has been added and several changes have been made to the exercise set.
- **5.4** The introduction to the wrapping function has been enhanced with a graphical representation. An Interactive Demonstration that involves the wrapping function has been added. Exercises that allow the student to graphically estimate the value of the wrapping function, as well as new application exercises, have been added.
- **5.5** Definition boxes that allow the student to easily find important concepts have been added. The new definition boxes now include simple examples of the definitions. Many examples have been revised so there is a consistent approach to graphing trigonometric functions.
- **5.6** Definition boxes that allow the student to easily find important concepts have been added. The new definition boxes now include simple examples of the definitions. Many examples have been revised so there is a consistent approach to graphing trigonometric functions. Application exercises have been added.
- **5.7** An Interactive Demonstration that involves the graphs of trigonometric functions has been added. An exercise involving the phenomenon of beats and other application exercises have been added.
- **5.8** New application exercises have been added.

Chapter 6 **Trigonometric Identities, Inverse Functions, and Equations**

- A new chapter opener introduces some of the concepts in this chapter.
- **6.3** A visual insight exercise that illustrates a half-angle identity has been added to the Enrichment Exercises.
- **6.6** The example that uses a sine regression to model the illumination of the moon has been updated. All sine regression application exercises have been updated.
- The Exploring Concepts with Technology feature has been revised to illustrate how WolframAlpha can be used to play musical tones and beats.
- The sine regression application exercises in the Chapter Review and the Chapter Test have been updated.

Chapter 7 **Applications of Trigonometry**

- **7.1** Additional introductory information concerning the ASA, AAS, and the SSA cases has been added.
- **7.3** The dot product example has been revised.
- The Exploring Concepts with Technology feature has been revised to illustrate how WolframAlpha can be used to perform vector operations and solve vector application problems.

Chapter 8 **Topics in Analytic Geometry**

- A new chapter opener introduces some of the concepts in this chapter.
- **8.1** The Integrating Technology feature of this section has been expanded to include instructions on how to use WolframAlpha to graph parabolas and to find the focus and vertex of a parabola. A new Interactive Demonstration illustrates the relationships between the standard form of the equation of a parabola and its graph. A 3-D optical illusion exercise has been added to the Enrichment Exercises.

- **8.2** The new Interactive Demonstration illustrates the relationships between the standard form of the equation of an ellipse and its graph. The Integrating Technology feature has been expanded to include instructions on how to use WolframAlpha to graph ellipses and to determine the foci, vertices, and center of an ellipse. Several application exercises have been added.
- **8.3** The new Interactive Demonstration illustrates the relationships between the standard form of the equation of a hyperbola and its graph. The Integrating Technology feature has been expanded to include instructions on how to use WolframAlpha to graph a hyperbola and to determine the foci, vertices, and center of a hyperbola. Several application exercises have been added.
- **8.4** A new example involving the rotation-of-axes formulas has been added. A new example illustrates how to use WolframAlpha to graph a conic.
- **8.5** Two new Integrating Technology features and two Enrichment Exercises have been added.
- **8.6** The figue that illustrates the focus-directrix definitions of the conics has been expanded to include an ellipse and a hyperbola.
- **8.7** Exercises that involve parametric equations in an *xyz*-coordinate system have been added.
- The Exploring Concepts with Technology feature now provides instructions on how to use our new Interactive Demonstration that involves conics and polar equations.
- Application exercises have been added to the Review Exercises and to the Chapter Test.

Chapter 9 Systems of Equations and Inequalities

- **9.1** The Integrating Technology feature has been expanded to include instructions on how to use WolframAlpha to solve systems of linear equations. Application exercises and Enrichment Exercises have been added.
- **9.2** Application exercises have been added.
- **9.3** The Integrating Technology feature has been expanded to include instructions on how to use WolframAlpha to solve nonlinear systems of equations. Enrichment Exercises have been added.
- The new Exploring Concepts with Technology feature illustrates how to use WolframAlpha to solve linear programming problems.

Chapter 10 Matrices

- **10.1** The Echelon Form Procedure and the Gaussian Elimination example have been revised. A new example and new exercises have been added.
- **10.2** A social network graph introduces the concept of adjacency matrices. A social networks exercise has been added to the Enrichment Exercises.
- **10.3** The Integrating Technology feature has been expanded to include instructions on how to use WolframAlpha to find the inverse of a matrix.
- **10.4** Cramer's Rule is now included in this section. Three examples have been added.
- New exercises have been added to the Review Exercises and to the Chapter Test.

Chapter 11 Sequences, Series, and Probability

- A new chapter opener introduces some of the concepts in this chapter.
- **11.1** New examples have been added.
- **11.2** A new example and new application exercises have been added.
- **11.5** A new example has been added.
- **11.6** Three examples have been revised and several application exercises have been added.
- **11.7** Two examples have been revised and several application exercises have been added.

Supplements

For the Instructor

Instructor Companion Website
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via www.cengage.com/login. Formerly found on the PowerLecture DVD, access and download PowerPoint presentations, images, and more.

Complete Solutions Manual for Aufmann/Nation's Algebra and Trigonometry, 8e
ISBN: 978-1-285-45107-7
(Included on the Instructor Companion Website)
The complete solutions manual provides worked-out solutions to all of the problems in the text.

Text-Specific DVDs for Aufmann/Nation's Algebra and Trigonometry Series, 8e
ISBN: 978-1-285-45882-3
Available to adopting instructors, these DVDs, which cover all sections in the text, are hosted by Dana Mosely and captioned for the hearing-impaired. Ideal for promoting individual study and review, these comprehensive DVDs also support students in online courses or may be checked out by a student who may have missed a lecture. 12 DVDs contain over 40 hours of video.

Enhanced WebAssign®
www.webassign.net
Online Access Code ISBN: 978-1-285-85831-9
Printed Access Card ISBN: 978-1-285-85833-3
Exclusively from Cengage Learning, Enhanced WebAssign® offers an extensive online program for Algebra and Trigonometry to encourage the practice that's so critical for concept mastery. The meticulously crafted pedagogy and exercises in this text become even more effective in Enhanced WebAssign, supplemented by multimedia tutorial support and immediate feedback as students complete their assignments. Algorithmic problems allow you to assign unique versions to each student. The Practice Another Version feature (activated at your discretion) allows students to attempt the questions with new sets of values until they feel confident enough to work the original problem. Students benefit from a new Premium eBook with highlighting and search features; Personal Study Plans (based on diagnostic quizzing) that identify chapter topics they still need to master; and links to video solutions, interactive tutorials, and even live online help.

Cognero
(Included on the Instructor Companion Website)
Cengage Learning Testing Powered by Cognero is a flexible, online system that allows you to author, edit, and manage test bank content from multiple Cengage Learning solutions; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want.

Solution Builder
www.cengage.com/solutionbuilder
This online instructor database offers complete worked-out solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class.

Syllabus Creator
(Included on the Instructor Companion Website)
Quickly and easily create your course syllabus with **Syllabus Creator**, which was created by the authors.

For the Student

Study Guide with Student Solutions Manual for Aufmann/Nation's Algebra and Trigonometry, 8e
ISBN: 978-1-285-45112-1
The student solutions manual reinforces student understanding and aids in test preparation with detailed explanations, worked-out examples, and practice problems. Lists key ideas to master and builds problem-solving skills. Includes worked solutions to the odd-numbered problems in the text.

Enhanced WebAssign®
www.webassign.net
Online Access Code ISBN: 978-1-285-85831-9
Printed Access Card ISBN: 978-1-285-85833-3
Exclusively from Cengage Learning, Enhanced WebAssign® offers an extensive online program for Algebra and Trigonometry to encourage the practice that's so critical for concept mastery. You'll receive multimedia tutorial support as you complete your assignments. You'll also benefit from a new Premium eBook with highlighting and search features; Personal Study Plans (based on diagnostic quizzing) that identify chapter topics you still need to master; and links to video solutions, interactive tutorials, and even live online help.

CengageBrain.com
Visit *www.cengagebrain.com* to access additional course materials and companion resources. At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where free companion resources can be found.

Acknowledgments

We would like to thank the wonderful team of editors, accuracy checkers, proofreaders, and solutions manual authors. Special thanks to Cindy Harvey and Lauri Semarne. Cindy Harvey was a very valuable asset during development and production of the manuscript. Lauri Semarne was the accuracy reviewer for both *College Algebra* and *Algebra and Trigonometry*, and wrote the solutions for the Complete Solutions Manual and the Student Solutions Manual for *College Algebra* and *Algebra and Trigonometry*. Lauri has improved the accuracy of the texts and provided valuable suggestions for improving the texts.

We are grateful to the users of the previous edition for their helpful suggestions on improving the text. Also, we sincerely appreciate the time, effort, and suggestions of the reviewers of this edition:

Francis Bourgeois—*Lone Star College*
Jennifer Byall—*Globe University*
John Daley—*Globe University/Minnesota School of Business*
Grant Fraser—*California State University, Los Angeles*
Gustave Milburn—*Montgomery College*
Jan Miller—*Bay Mills Community College*
Daniel Roddin—*Minnesota School of Business*
Kristina Sampson—*Lone Star College*
Mark Walker—*University of Nebraska, Lincoln*
Edward Watkins—*Florida State College*


© Eric Broder Van Dyke/Shutterstock.com

CHAPTER P

Preliminary Concepts

Sabermetrics

The film *Moneyball* was based on the book *Moneyball: The Art of Winning an Unfair Game* by Michael Lewis. It recounts the true story of how the Oakland Athletics baseball team used mathematics to select players for its team. They used what has become known as **sabermetrics,** introduced by Bill James, to objectively evaluate a player's performance using mathematics.

> Bill James defined sabermetrics as "the search for objective knowledge about baseball." Thus, sabermetrics attempts to answer objective questions about baseball, such as "which player on the Red Sox contributed the most to the team's offense?" or "How many home runs will Miguel Cabrera hit next year?" It cannot deal with the subjective judgments which are also important to the game, such as "Who is your favorite player?" or "That was a great game."[1]

In sabermetrics, a SLOB is not a bad thing. Instead, a SLOB is one of the measures of a player's performance. SLOB stands for "**sl**ugging times **o**n **b**ase average." A SLOB value of 0.3 is considered very good. For instance, Lou Gehrig had a SLOB value of 0.283. Many of the sabermetric measures are based on ratios such as the expressions given in Exercises 129 and 130 on page 16.

[1] David J. Grabiner, "The Sabermetric Manifesto," *The Baseball Archive*. Available online at http://remarque.org/~grabiner/manifesto.txt

SECTION P.1

The Real Number System

Sets

Human beings share the desire to organize and classify. Astronomers classify stars by such characteristics as color, mass, size, temperature, and distance from Earth. Mathematicians likewise place objects with similar properties in *sets*. A **set** is a collection of objects. The objects are called **elements** of the set. Sets are denoted by placing braces around the elements in the set.

The numbers that we use to count things, such as the number of books in a library or the number of songs in a music collection, are called the **natural numbers.**

$$\textbf{Natural numbers} = \{1, 2, 3, 4, 5, 6, \ldots\}$$

Each natural number greater than 1 is a *prime* number or a *composite* number. A **prime number** is a natural number greater than 1 that is divisible (evenly) only by itself and 1. For example, 2, 3, 5, 7, 11, and 13 are the first six prime numbers. A natural number, other than 1, that is not a prime number is a **composite number.** The numbers 4, 6, 8, and 9 are the first four composite numbers. Note that each of these numbers is divisible by a number other than itself and 1. For instance, 8 is divisible by 1, 2, 4, and 8.

The whole numbers include zero and the natural numbers.

$$\textbf{Whole numbers} = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

We also need numbers to measure temperature below zero or, in accounting, when a company incurs a loss.

$$\textbf{Integers} = \{\ldots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \ldots\}$$

The integers $\ldots, -6, -5, -4, -3, -2, -1$ are **negative integers.** The integers $1, 2, 3, 4, 5, 6, \ldots$ are **positive integers** (or natural numbers). The integer 0 is neither a positive nor a negative integer.

Still other numbers are needed to designate part of a whole, such as a screw that is three-fourths inch long.

$$\textbf{Rational numbers} = \left\{\frac{p}{q}, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0\right\}$$

The numbers $\frac{3}{4}$, $-\frac{9}{2}$, and $\frac{7}{1}$ are examples of rational numbers. Note that $\frac{7}{1} = 7$. Because any integer n can be written with a denominator of $1\left(n = \frac{n}{1}\right)$, **all integers are rational numbers.**

A rational number written as a fraction can be written as a decimal by dividing the numerator by the denominator. As shown below, the result is either a **terminating decimal** such as 0.45 or a **repeating decimal** such as $0.218181818\ldots$, where the digits 18 are continually repeated. In this case, we frequently place a bar over the repeating digits and write $0.2181818\ldots = 0.2\overline{18}$.

$$\begin{array}{r} 0.45 \\ 20\overline{)\,9.00} \\ \underline{-8\,0} \\ 1\,00 \\ \underline{-1\,00} \\ 0 \end{array}$$

This is a terminating decimal. The remainder is zero.

$$\frac{9}{20} = 0.45$$

$$\begin{array}{r} 0.21818 \\ 55\overline{)\,12.00000} \\ \underline{-11\,0} \\ 1\,00 \\ \underline{-55} \\ 450 \\ \underline{-440} \\ 100 \\ \underline{-55} \\ 450 \\ \underline{-440} \\ 10 \end{array}$$

This is a repeating decimal. Note that the remainders 10 and 45 are repeating. The remainder is never zero.

$$\frac{12}{55} = 0.2\overline{18}$$

Math Matters

Archimedes (c. 287–212 B.C.E.) was the first to calculate π with any degree of precision. He was able to show that

$$3\frac{10}{71} < \pi < 3\frac{1}{7}$$

from which we get the approximation

$$3\frac{1}{7} = \frac{22}{7} \approx \pi$$

The use of the symbol π for this quantity was introduced by Leonhard Euler (1707–1783) in 1739, approximately 2000 years after Archimedes.

Math Matters

Sophie Germain (1776–1831) was born in Paris, France. Because enrollment in the university she wanted to attend was available only to men, Germain attended under the name of Antoine-August Le Blanc. Eventually her ruse was discovered, but not before she came to the attention of Pierre Lagrange, one of the best mathematicians of the time. He encouraged her work and became a mentor to her. A certain type of prime number is named after her, called a *Germain prime number*. It is a number p such that p and $2p + 1$ are both prime. For instance, 11 is a Germain prime because $2(11) + 1 = 23$, and 11 and 23 are both prime numbers. Germain primes are used in public key cryptography, a method used to send secure communications over the Internet.

Numbers that are not rational numbers are called **irrational numbers.** In decimal form, an irrational number has a decimal representation that never terminates nor repeats. One of the best known irrational numbers is pi, denoted by the Greek letter π. An approximate value of π is 3.14592654. . . . Other examples of irrational numbers are 2.13113111311113. . . and the square root of any prime number such as $\sqrt{11} \approx 3.31662479\ldots$. The rational numbers and irrational numbers taken together are the **real numbers.**

The relationships among the various sets of numbers are shown in Figure P.1.

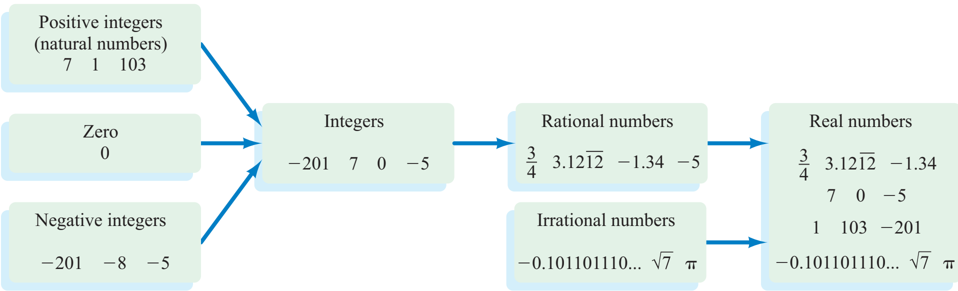


Figure P.1

EXAMPLE 1 Classify Real Numbers

Determine which of the following numbers are

a. integers **b.** rational numbers **c.** irrational numbers

d. real numbers **e.** prime numbers **f.** composite numbers

$$-0.2, \quad 0, \quad 0.\overline{3}, \quad 0.71771777177771\ldots, \quad \pi, \quad 6, \quad 7, \quad 41, \quad 51$$

Solution

a. Integers: 0, 6, 7, 41, 51

b. Rational numbers: $-0.2, 0, 0.\overline{3}, 6, 7, 41, 51$

c. Irrational numbers: $0.71771777177771\ldots, \pi$

d. Real numbers: $-0.2, 0, 0.\overline{3}, 0.71771777177771\ldots, \pi, 6, 7, 41, 51$

e. Prime numbers: 7, 41

f. Composite numbers: 6, 51

Try Exercise 8, page 14

Each member of a set is called an **element** of the set. For instance, if $C = \{2, 3, 5\}$, then the elements of C are 2, 3, and 5. The notation $2 \in C$ is read "2 is an element of C." Set A is a **subset** of set B if every element of A is also an element of B, and we write $A \subseteq B$. For instance, the set of negative integers $\{-1, -2, -3, -4, \ldots\}$ is a subset of the set of integers. The set of positive integers $\{1, 2, 3, 4, \ldots\}$ (the natural numbers) is also a subset of the set of integers.

Note

The order of the elements of a set is not important. For instance, the set of natural numbers less than 6 given at the right could have been written {3, 5, 2, 1, 4}. It is customary, however, to list elements of a set in numerical order.

Question • Are the integers a subset of the rational numbers?

The **empty set**, or **null set**, is the set that contains no elements. The symbol $\varnothing$ is used to represent the empty set. The set of people who have run a 2-minute mile is the empty set.

Answer • Yes.

The set of natural numbers less than 6 is $\{1, 2, 3, 4, 5\}$. This is an example of a **finite set**; all the elements of the set can be listed. The set of all natural numbers is an example of an **infinite set**. There is no largest natural number, so all the elements of the set of natural numbers cannot be listed.

Sets are often written using **set-builder notation**. Set-builder notation can be used to describe almost any set, but it is especially useful when writing infinite sets. For instance, the set

$$\{2n \mid n \in \text{natural numbers}\}$$

is read as "the set of elements $2n$ such that n is a natural number." By replacing n with each of the natural numbers, we obtain the set of positive even integers: $\{2, 4, 6, 8, \ldots\}$.

The set of real numbers greater than 2 is written

$$\{x \mid x > 2, x \in \text{real numbers}\}$$

and is read "the set of x such that x is greater than 2 and x is an element of the real numbers."

Much of the work we do in this text uses the real numbers. With this in mind, we will frequently write, for instance, $\{x \mid x > 2, x \in \text{real numbers}\}$ in a shortened form as $\{x \mid x > 2\}$, where we assume that x is a real number.

Math Matters

A **fuzzy set** is one in which each element is given a "degree" of membership. The concepts behind fuzzy sets are used in a wide variety of applications such as traffic lights, washing machines, and computer speech recognition programs.

EXAMPLE 2 Use Set-Builder Notation

List the four smallest elements in $\{n^3 \mid n \in \text{natural numbers}\}$.

Solution

Because we want the four *smallest* elements, we choose the four smallest natural numbers. Thus $n = 1, 2, 3$, and 4. Therefore, the four smallest elements of the set $\{n^3 \mid n \in \text{natural numbers}\}$ are 1, 8, 27, and 64.

Try Exercise 12, page 14

Union and Intersection of Sets

Just as operations such as addition and multiplication are performed on real numbers, operations are performed on sets. Two operations performed on sets are union and intersection. The union of two sets A and B is the set of elements that belong to A or to B, or to both A and B.

Definition of the Union of Two Sets

The **union** of two sets, written $A \cup B$, is the set of all elements that belong to either A or B. In set-builder notation, this is written

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

EXAMPLE

Given $A = \{2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3, 4\}$, find $A \cup B$.

$A \cup B = \{0, 1, 2, 3, 4, 5\}$ • Note that an element that belongs to both sets is listed only once.

The intersection of the two sets A and B is the set of elements that belong to both A and B.

Definition of the Intersection of Two Sets

The **intersection** of two sets, written $A \cap B$, is the set of all elements that are common to both A and B. In set-builder notation, this is written

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

EXAMPLE

Given $A = \{2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3, 4\}$, find $A \cap B$.

$A \cap B = \{2, 3, 4\}$ • The intersection of two sets contains the elements common to both sets.

If the intersection of two sets is the empty set, the two sets are said to be **disjoint.** For example, if $A = \{2, 3, 4\}$ and $B = \{7, 8\}$, then $A \cap B = \varnothing$ and A and B are disjoint sets.

EXAMPLE 3 Find the Union and Intersection of Sets

Find each intersection or union given $A = \{0, 2, 4, 6, 10, 12\}$, $B = \{0, 3, 6, 12, 15\}$, and $C = \{1, 2, 3, 4, 5, 6, 7\}$.

a. $A \cup C$ **b.** $B \cap C$ **c.** $A \cap (B \cup C)$ **d.** $B \cup (A \cap C)$

Solution

a. $A \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 10, 12\}$ • The elements that belong to A or C

b. $B \cap C = \{3, 6\}$ • The elements that belong to B and C

c. First, determine $B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 12, 15\}$. Then
$A \cap (B \cup C) = \{0, 2, 4, 6, 12\}$ • The elements that belong to A and $(B \cup C)$

d. First, determine $A \cap C = \{2, 4, 6\}$. Then
$B \cup (A \cap C) = \{0, 2, 3, 4, 6, 12, 15\}$ • The elements that belong to B or $(A \cap C)$

Try Exercise 22, page 14

Interval Notation

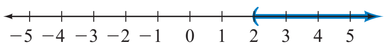


Figure P.2

The graph of $\{x \mid x > 2\}$ is shown in Figure P.2. The set is the real numbers greater than 2. The parenthesis at 2 indicates that 2 is not included in the set. Rather than write this set of real numbers using set-builder notation, we can write the set in **interval notation** as $(2, \infty)$.

In general, the interval notation

(a, b) represents all real numbers between a and b, not including a and b. This is an **open interval.** In set-builder notation, we write $\{x \mid a < x < b\}$. The graph of $(-4, 2)$ is shown in Figure P.3.

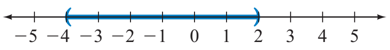


Figure P.3

$[a, b]$ represents all real numbers between a and b, including a and b. This is a **closed interval.** In set-builder notation, we write $\{x \mid a \le x \le b\}$. The graph of $[0, 4]$ is shown in Figure P.4. The brackets at 0 and 4 indicate that those numbers are included in the graph.

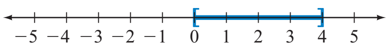


Figure P.4

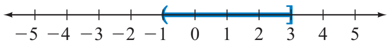


Figure P.5

$(a, b]$ represents all real numbers between a and b, not including a but including b. This is a **half-open interval.** In set-builder notation, we write $\{x \mid a < x \leq b\}$. The graph of $(-1, 3]$ is shown in Figure P.5.

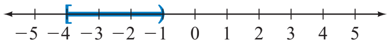
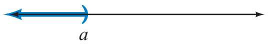


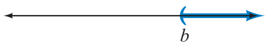
Figure P.6

$[a, b)$ represents all real numbers between a and b, including a but not including b. This is a **half-open interval.** In set-builder notation, we write $\{x \mid a \leq x < b\}$. The graph of $[-4, -1)$ is shown in Figure P.6.

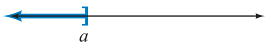
Subsets of the real numbers whose graphs extend forever in one or both directions can be represented by interval notation using the **infinity symbol** ∞ or the **negative infinity symbol** $-\infty$.



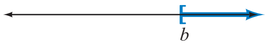
$(-\infty, a)$ represents all real numbers less than a.



(b, ∞) represents all real numbers greater than b.



$(-\infty, a]$ represents all real numbers less than or equal to a.



$[b, \infty)$ represents all real numbers greater than or equal to b.

$(-\infty, \infty)$ represents all real numbers.

EXAMPLE 4 Graph a Set Given in Interval Notation

Graph $(-\infty, 3]$. Write the interval in set-builder notation.

Solution

The set is the real numbers less than or equal to 3. In set-builder notation, this is the set $\{x \mid x \leq 3\}$. Draw a right bracket at 3, and darken the number line to the left of 3, as shown in Figure P.7.

Figure P.7

▶ Try Exercise 40, page 14

Caution

It is *never* correct to use a bracket when using the infinity symbol. For instance, $[-\infty, 3]$ is not correct. Nor is $[2, \infty]$ correct. Neither negative infinity nor positive infinity is a real number and therefore cannot be contained in a closed interval.

The set $\{x \mid x \leq -2\} \cup \{x \mid x > 3\}$ is the set of real numbers that are either less than or equal to -2 *or* greater than 3. We also could write this in interval notation as $(-\infty, -2] \cup (3, \infty)$. The graph of the set is shown in Figure P.8.

The set $\{x \mid x > -4\} \cap \{x \mid x < 1\}$ is the set of real numbers that are greater than -4 *and* less than 1. Note from Figure P.9 that this set is the interval $(-4, 1)$, which can be written in set-builder notation as $\{x \mid -4 < x < 1\}$.

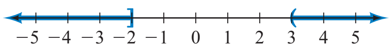


Figure P.8

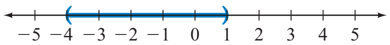


Figure P.9

EXAMPLE 5 Graph Intervals

Graph the following. Write **a.** and **b.** using interval notation. Write **c.** and **d.** using set-builder notation.

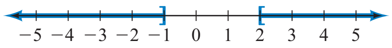
a. $\{x \mid x \leq -1\} \cup \{x \mid x \geq 2\}$

b. $\{x \mid x \geq -1\} \cap \{x \mid x < 5\}$

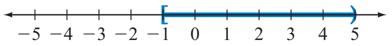
c. $(-\infty, 0) \cup [1, 3]$

d. $[-1, 3] \cap (1, 5)$

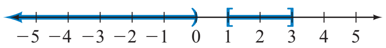
Solution

a. 

$(-\infty, -1] \cup [2, \infty)$

b. 

$[-1, 5)$

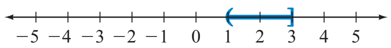
c. 

$\{x \mid x < 0\} \cup \{x \mid 1 \le x \le 3\}$

d. The graphs of $[-1, 3]$, in red, and $(1, 5)$, in blue, are shown below.

Note that the intersection of the sets occurs where the graphs intersect. Although $1 \in [-1, 3]$, $1 \notin (1, 5)$. Therefore, 1 does not belong to the intersection of the sets. On the other hand, $3 \in [-1, 3]$ and $3 \in (1, 5)$. Therefore, 3 belongs to the intersection of the sets.

Thus we have the following.



$\{x \mid 1 < x \le 3\}$

Try Exercise 50, page 14

Absolute Value and Distance

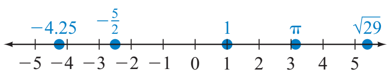


Figure P.10

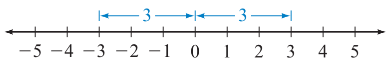


Figure P.11

The real numbers can be represented geometrically by a **coordinate axis** called a **real number line**. Figure P.10 shows a portion of a real number line. The number associated with a point on a real number line is called the **coordinate** of the point. The point corresponding to zero is called the **origin**. Every real number corresponds to a point on the number line, and every point on the number line corresponds to a real number.

The *absolute value* of a real number a, denoted $|a|$, is the distance between a and 0 on the number line. For instance, $|3| = 3$ and $|-3| = 3$ because both 3 and -3 are 3 units from zero. See Figure P.11.

In general, if $a \ge 0$, then $|a| = a$; however, if $a < 0$, then $|a| = -a$ because $-a$ is positive when $a < 0$. This leads to the following definition.

> ***Note***
>
> The second part of the definition of absolute value states that if $a < 0$, then $|a| = -a$. For instance, if $a = -4$, then $|a| = |-4| = -(-4) = 4$.

Definition of Absolute Value

The **absolute value** of the real number a is defined by

$$|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$$

EXAMPLE

$|5| = 5$ $\qquad |-4| = 4$ $\qquad |0| = 0$

EXAMPLE 6 Simplify an Absolute Value Expression

Simplify $|x + 4| - |2x - 6|$ given that $-3 \le x \le 2$.

Solution

Recall that $|a| = -a$ when $a < 0$ and $|a| = a$ when $a \ge 0$.

(continued)

When $-3 \le x \le 2$, $x + 4 > 0$ and $2x - 6 < 0$. Therefore, $|x + 4| = x + 4$ and

$|2x - 6| = -(2x - 6)$. Thus

$$\begin{aligned} |x + 4| - |2x - 6| &= (x + 4) - [-(2x - 6)] \\ &= (x + 4) + (2x - 6) \\ &= 3x - 2 \end{aligned}$$

Try Exercise 60, page 14

The definition of *distance* between two points on a real number line makes use of absolute value.

Definition of the Distance Between Points on a Real Number Line

If a and b are the coordinates of two points on a real number line, the **distance** between the graph of a and the graph of b, denoted by $d(a, b)$, is given by $d(a, b) = |a - b|$.

EXAMPLE

Find the distance between the point whose coordinate on the real number line is -2 and the point whose coordinate is 5.

$$d(-2, 5) = |-2 - 5| = |-7| = 7$$

Note in Figure P.12 that there are 7 units between -2 and 5. Also note that the order of the coordinates in the formula does not matter.

$$d(5, -2) = |5 - (-2)| = |7| = 7$$

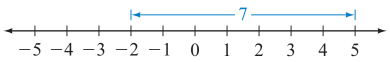


Figure P.12

EXAMPLE 7 Use Absolute Value to Express the Distance Between Two Points

Express the distance between a and -3 on the number line using absolute value notation.

Solution

$$d(a, -3) = |a - (-3)| = |a + 3|$$

Try Exercise 70, page 15

> ***Math Matters***
>
> The expression 10^{100} is called a *googol*. The term was coined by the 9-year-old nephew of the American mathematician Edward Kasner. Many calculators do not provide for numbers of this magnitude, but it is no serious loss. To appreciate the magnitude of a googol, consider that if all the atoms in the known universe were counted, the number would not even be close to a googol. But if a googol is too small for you, try 10^{googol}, which is called a *googolplex*. As a final note, the name of the Internet site Google.com is a takeoff on the word *googol*.

Exponential Expressions

A compact method of writing $5 \cdot 5 \cdot 5 \cdot 5$ is 5^4. The expression 5^4 is written in **exponential notation.** Similarly, we can write

$$\frac{2x}{3} \cdot \frac{2x}{3} \cdot \frac{2x}{3} \quad \text{as} \quad \left(\frac{2x}{3}\right)^3$$

Exponential notation can be used to express the product of any expression that is used repeatedly as a factor.

Definition of Natural Number Exponents

If b is any real number and n is a natural number, then

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ is a factor } n \text{ times}}$$

where b is the **base** and n is the **exponent**.

EXAMPLE

$$\left(\frac{3}{4}\right)^3 = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$$

$$-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5) = -625$$

$$(-5)^4 = (-5)(-5)(-5)(-5) = 625$$

Pay close attention to the difference between -5^4 (the base is 5) and $(-5)^4$ (the base is -5).

EXAMPLE 8 Evaluate an Exponential Expression

Evaluate.

a. $(-3^4)(-4)^2$ **b.** $\dfrac{-4^4}{(-4)^4}$

Solution

a. $(-3^4)(-4)^2 = -(3 \cdot 3 \cdot 3 \cdot 3) \cdot (-4)(-4) = -81 \cdot 16 = -1296$

b. $\dfrac{-4^4}{(-4)^4} = \dfrac{-(4 \cdot 4 \cdot 4 \cdot 4)}{(-4)(-4)(-4)(-4)} = \dfrac{-256}{256} = -1$

Try Exercise 76, page 15

Order of Operations Agreement

The approximate pressure p, in pounds per square inch, on a scuba diver x feet below the water's surface is given by

$$p = 15 + 0.5x$$

The pressure on the diver at various depths is given below.

10 feet	$15 + 0.5(10) = 15 + 5 = 20$ pounds
20 feet	$15 + 0.5(20) = 15 + 10 = 25$ pounds
40 feet	$15 + 0.5(40) = 15 + 20 = 35$ pounds
70 feet	$15 + 0.5(70) = 15 + 35 = 50$ pounds

Note that the expression $15 + 0.5(70)$ has two operations, addition and multiplication. When an expression contains more than one operation, the operations must be performed in a specified order, as given by the Order of Operations Agreement.